Physical Simulation of Adhesive Bodies and Their Interactions with Their Environments

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May 2020
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1 Abstract

This paper outlines modifications that can be made to standard cloth and rigid body algorithms to simulate tape. We discuss how to make one-sided and two-sided tape, which we will refer to as adhesive bodies, interact with an environment consisting of static bodies, rigid bodies, and adhesive bodies. In this paper, we cover criteria for the sticking and detachment of tape to other faces in the environment, and we give forces that should act on the faces to create behavior that simulates tape. These forces include a force that resists sliding of the adhesive body over other bodies and an adhesive force that resists the peeling of the adhesive body off of another body.

2 Introduction

Tape is a common material seen throughout our lives. We use it to make quick fixes in our houses due to its strength and flexibility. On one side, tape is non-adhesive and acts similarly to cloth, while on the other side, the tape is adhesive and will stick to whatever it comes in contact with.

The research presented by this paper extends models of cloth simulation to simulate tape. First, we cover a mathematical basis to build our discretization off of. Next, we describe the objects in our scenes and what precomputation must be done. After that, we discuss forces and impulses that we use to model adhesive bodies and their interactions with their environments. Next, we discuss criteria for when faces on the adhesive bodies should attach or detach from environmental faces. Finally, we show how the adhesive bodies evolve over time in test simulations.
3 Literature Review

Our first step in deciding on how to simulate tape was to survey the field and see what work had previously been done. Barthel provides an overview of adhesive modelling in the 75 years leading up to 2005 from models like JKR and DMT to more complex models [2]. Popov et al. [5] show that the shape of a rigid punch on an infinite adhesive plane will detach first at outer sharp corners and form a circular front. Kendall [4] describes the surface and elastic energy of a rigid disc with an infinite elastic surface and we will use the surface energy provided in the paper. The paper by Azizinaghsh and Ghaednia [1] gives a numerical model for removing a piece of tape from a horizontal surface where the tape’s bonds are modelled with vertical springs for adhesion and horizontal springs for the tape’s backing. This is similar to the model we propose except we do not model our adhesion with pure springs and have a tangential force. Rey et al. [6] discuss simulating the adhesive force on rough surfaces in the case where both objects involved in the interaction can not be deformable. Our model does not account for the roughness of objects but does extend to interactions where both the tape and the object being stuck to are deformable.

4 Basic Model

Following the Adhesive Paper by Kendall [4] our goal was to create a simple mathematical model for adhesivity. Let us consider an adhered region whose boundary can be represented as a closed curve in two-dimensional space, we will now look at the energy of such a closed region surrounded by a boundary \( B \). We let \( B \) be a map from \([0, 1]\) to \( \mathbb{R}^2 \) with \( B(0) = B(1) \) such that \( B \) encloses the adhered region in a clockwise orientation. Generalizing the equations of the paper and ignoring the elastic energy by assuming the material is perfectly inelastic we get that the surface energy should be proportional to the contact area. We also note that the energy should be proportional to \( \gamma \) which represents how much energy is required to make it such that a region of unit area is no longer adhered. A higher \( \gamma \) value means that the tape is more sticky while a lower \( \gamma \) value means that the tape will not stick as well. This allows us to derive the following formula for the surface energy:

\[
E_s = \int_{r=0}^{1} \gamma \dot{B}_x(r) B_y(r) dr
\]

where \( \dot{B} \) is the time derivative of \( B \), and coordinates for the subscripts mean to take that component.

We will want to discretize the above energy for triangle meshes. The first thing we will do then is replace the above integral with a summation of the force per face. We then get that:

\[
E_s = \sum_{f \in F} \int_{f} \int_{f} -\gamma dA
\]

which is simply adding up a contribution of \(-\gamma\) per unit area on each triangle face.
The above is the energy when the tape is fully adhered to a region, we would like to make an adhesive energy that varies depending on the distance of the face from the object it is adhered to. This adhesive energy should be zero when the two objects are far apart and \( E_s \) when the objects are in full contact.

Let \( u \) and \( v \) be a parametrization of each triangular face and let \( D(u, v) \) be the difference vector from the point at \((u, v)\) to the closest point on the adhered region. We will let \( D_n(v) \) of a vector \( v \) denote the length of \( v \) in the normal direction of the adhered region. Let \( G(x) \) be a continuous and differentiable function that represents how much contact there should be at a distance \( x \) taking on values between zero for no contact and one for full contact. We then note we can write an adhesive energy as follows:

\[
E_{\text{adhesive}} = \sum_{f \in F} \int \int_{f} -\gamma G(D_n(D(u, v))) dA
\]

Computing these integrals per time step in a simulation would be slow, so instead we choose to approximate the above using two techniques. First, we choose to use the centroid of a triangular face as a representative of the whole face’s area. Second, for a triangular face \( f \) we introduce a notion of a stuck point \( s_f \) that represents a point that was closest to \( f \) and does not change until detachment criteria are met. These two techniques allow us to avoid an integral over the face along with not having to scan the environment for the closest point at every time step. With these approximations in mind and letting \( A \) be the area of the adhered region we can write the above as:

\[
E_{\text{adhesive}} = \sum_{f \in F} -\gamma A G(D_n(f_{\text{centroid}} - s_f))
\]

This energy does not resist the tape sliding parallel to the adhered region since such sliding does not vary \( D_n \). However, tape should not be able to freely slide across a surface so we need to have an additional energy in place to prevent this. We let \( D_t(v) \) for a vector \( v \) return the part of the vector that is parallel to the adhered region. The notion of the stuck point \( s_f \) we constructed earlier allows us to model a resistance to sliding by trying to keep the face \( f \) close to the \( s_f \). We then use a spring potential based on the parallel distance to keep the tape point from sliding without resistance:

\[
E_{\text{slide}} = \sum_{f \in F} \zeta A ||D_t(f_{\text{centroid}} - s_f)||^2
\]

where \( \zeta \) is a friction coefficient representing how much the tape should resist sliding. Since we will allow the stuck point to change if there is too much strain in \( D_t \) this will create an effect that damps energy if tape tries to slide across a surface.

These energies are the basic mathematical foundation of the work we will do in section 10. In that section we will go into detail on how to calculate \( s_f, f_{\text{centroid}}, A, D_t, \) and \( D_n \) along with selecting a choice for a \( G \) function in 10.6.
5 Objects in Scenes

5.1 Overview

The simulations described in this paper contain three types of bodies.

- Static bodies are stationary objects that will not move throughout the simulation. They are defined by a base triangle mesh.
- Rigid Bodies in our simulation are bodies defined with a template triangle mesh that can be translated and rotated.
- Adhesive Bodies are two-dimensional surfaces defined by a base triangle mesh that can be deformed.

5.2 Rigid Bodies

For a rigid body based on a template triangle mesh $M$, we will have its configuration space be $q = \begin{bmatrix} \tau \\ \theta \end{bmatrix}$ with $\tau \in \mathbb{R}^3$ representing the translation of the rigid body and $\theta \in \mathbb{R}^3$ representing the rotation of the rigid body in axis-angle representation. For any vertex with local position $\bar{v}$ in $M$ we note we can convert the position into world coordinates by using $\phi_p$ where $\phi_p$ is defined as:

$$\phi_p(\bar{v}) = \text{rot}(\theta)\bar{v} + \tau$$

where $\text{rot}$ is the function that takes in an axis-angle representation and returns the corresponding rotation matrix. Similarly if we have a vector $\bar{n}$ in the local coordinates of $M$ we note we can convert it to world coordinates by using $\phi_v$ where $\phi_v$ is defined as:

$$\phi_v(\bar{n}) = \text{rot}(\theta)\bar{n}$$

5.3 Adhesive Bodies

An adhesive body based on a triangle mesh $M$ representing a two-dimensional surface with $n$ vertices has configuration space $q = [p_1, p_2, ..., p_{n-1}, p_n]^T$ with each $p_j$ representing the position of the $j$’th point as a vector in $\mathbb{R}^3$. The adhesive bodies in the simulation are two-sided and each side can either be sticky or nonstick. We define a local coordinate system for the adhesive body that is a pairing of a face index and barycentric coordinates. If $M$ has $f$ faces then the face index $\bar{i}$ can be chosen from $[1, f]$ and the barycentric coordinates $\bar{v}$ must lie in $\mathbb{R}^3$ with each component non-negative and all of the components summing to one. Letting $f_{j,1}$, $f_{j,2}$, and $f_{j,3}$ represent the indices of the vertices of the $j$’th face, we note we can convert from local coordinates $(\bar{i}, \bar{v})$ to world coordinates with $\mathbb{B}$ which is defined as:

$$\mathbb{B}(\bar{i}, \bar{v}) = [p_{f_{i,1}} \ p_{f_{i,2}} \ p_{f_{i,3}}] \bar{v}$$

We will use these barycentric coordinates later to represent local stuck points on adhesive bodies and deal with cloth-cloth collisions.
6 Precomputation

In this section, we will discuss the computations we run and store for an adhesive body when loading it up to run in a simulation. For each face $F$ of the triangle mesh we are using as an adhesive body, we will want to calculate and store its rest area.

We can calculate the rest area of the face made up of points with positions $a$, $b$, and $c$ with the following calculation:

$$\frac{1}{2} \| (b - a) \times (c - a) \|$$

Since $(b - a)$ and $(c - a)$ represent vectors that are the edges of the triangle face, their cross product represents the area of the parallelogram they form, and the face $F$ is itself half of that parallelogram.

We will also want to do some precomputation on the vertices of the adhesive body. We will be using a lumped mass model for our simulation of cloth, so we will want to assign a mass to each vertex. Every vertex is given a mass proportional to a third of the sum of the areas of faces the vertex is a part of.

During this precomputation step, we will also want to place springs on the adhesive body to keep the mesh from stretching or compressing too much. We will place a spring if two vertices $v_1$ and $v_2$ have an edge between them on the triangle mesh. Additionally, we will place a spring if $v_1$ and $v_2$ lie on opposite sides of an interior edge $e$ such that they create an anti-diagonal for the edge $e$.

7 Overview of Simulation

In this section we will give a high-level overview of how the simulation will run. First, we update the configurations of the rigid bodies and adhesive bodies using their velocities from the previous time step. Next, we calculate all the forces that should be applied to rigid bodies and adhesive bodies as described in sections 8 and 10. Then, we update the velocity of the adhesive bodies using the forces we calculated for them. After that, we do a check to see if any vertices on the adhesive bodies should stick or detach as described in section 9. Next, we use the rigid body forces to update their velocities. Finally, we check to see if any of the adhesive bodies are going to collide and deal with the collisions is so as described in section 11.

8 Forces on Adhesive Bodies

8.1 Gravity

The tape is affected by a gravity force which applies a user defined acceleration $g$ in the Y direction for each vertex. We iterate through each point in the adhesive body mesh and apply a force of $gm$ in the Y direction for a point with mass $m$. 
8.2 Springs

We use classic spring forces in our simulation of tape. Let $S$ be a spring between points $p_1$ and $p_2$ then we define the rest length $L$ to be the distance between the points in the template mesh. We also let $k$ be a user-defined parameter describing how stiff the springs in the adhesive body should be. Then we note we can describe the force that $s$ exerts on $p_1$ as:

$$-k(||p_1 - p_2|| - L)(p_1 - p_2)$$

and the force that $s$ exerts on $p_2$ as:

$$k(||p_1 - p_2|| - L)(p_1 - p_2)$$

8.3 Bending Force

We use the bending force method for triangular meshes given by Wang et al. [7]. We start with the cotangent Laplacian $L$ for our matrix. Then we will let $N$ be a matrix that is nonzero only for each row corresponding to a vertex c on the boundary with neighbors a and e also on the boundary. Let b be the third vertex on the face with a and c and let d be the third vertex on the face with c and e, then we note that the $i$’th row of $N$ is:

$$N_i x = \frac{\cot(\angle ced)}{2}(x_c - x_d) + \frac{\cot(\angle dce)}{2}(x_e - x_d) + \frac{\cot(\angle bac)}{2}(x_c - x_b) + \frac{\cot(\angle acb)}{2}(x_a - x_b)$$

We then take $K = L + N$ and then for our bending force for a bending stiffness $\beta$, mass matrix $M$, and configuration $q$ is:

$$F_{bending} = -\beta q^T K^T M^{-1} K$$

8.4 Slide Resist Force

The tape model has a force that resists the tape being slid along the surface it is stuck to. This force and its corresponding potential are discussed in detail in a later section.

8.5 Adhesive Force

We have an adhesive force that resists the taped being peeled away from the surface it is stuck onto. This force and its corresponding potential are discussed in detail in a later section.
9 Sticking and Detachment

Our discretization is set up such that a face on an adhesive body is either stuck to a single point on some object in the scene or is detached from all objects. In this section we will discuss criteria for determining when a face should go from being detached to being stuck or go from being stuck to an object to being detached.

9.1 Sticking

Let \( r_m \) be a constant that denotes our tape’s preferred distance to the stuck object representing the thickness of the tape. We say a tape face will stick to another face if its centroid is within \( 2r_m \) of another face and the side of the tape face that is facing towards the target face is sticky. At each simulation time step each face on the adhesive body will try to find all other faces that are within \( 2r_m \) of its centroid by using a bounding volume hierarchy. We will then go through the list of faces that are within this range and prune out any that are unreasonable—faces that are facing away from the tape and the tape face itself. We can detect which side of the tape is facing towards the target face by taking the dot product of the face normal with a vector from the tape face to the target face. If this dot product is positive then we are trying to stick to the front of the face; however, if the dot product is negative then we are trying to stick to the back of the face. We will then go through all of the faces in this list and find which one is the closest and calculate which point on that face is the closest. When sticking to a face we set a flag on the face saying that the face is stuck and record the position and normal of the face we are stuck to in the object’s local coordinates. We let a tape face stick to up to one object on its front if the front is sticky and up to one object on its back if the back is sticky.

9.2 Detachment

For a tape face \( T \) with face normal \( n_T \) stuck to another face \( F \) with face normal \( n_F \) at world point \( p \) we define a criteria for detachment. We have three criteria for when a tape face can detach from the region it is adhered to: if the faces are not well-aligned, if they slide too far apart, or if they get pulled too far apart. We determine that two faces are not well-aligned if the face \( T \)'s shadow on \( F \) defined by \( A_{\text{shadow}} \) is negative. We note if \( A_{\text{shadow}} \) evaluates to be negative this means that the two face normals roughly point in the same direction meaning it does not make physical sense for them to continue to be stuck together. We give a precise definition of \( A_{\text{shadow}} \) in section 10. To check if the faces have slid too far apart we...
first evaluate the tangent distance $D_t$ (defined in section 10) on the configuration to see if its value exceeds some user-defined constant and detach if so. For the final criteria we check to see if the distance in the stuck face’s normal direction $D_n$ (defined in section 10) on the configuration exceeds some user-defined constant and detach if so.

9.3 Behavior

We note that the sudden attachment or detachment of a tape face to an object can cause changes in the face’s total energy. For example, a tape face sliding along a surface and getting detached can decrease the adhesive energy suddenly. Alternatively, it could also detach and then end up instantly sticking to a face that is closer in the normal direction than the previous face causing the face’s energy to increase. In practice, the only effect we have noticed is that faces will sometimes pop out a bit when they were closer than their preferred distances.

10 Slide Resist and Adhesive Forces

In this section we will be discussing the two forces that will make the adhesive bodies act like tape along with the respective potentials for the forces. These forces only act on a tape face when it is stuck to an object as defined in section 9. We will discuss an adhesive force that resists peeling and acts along the normal direction of the face the tape is stuck to and a force that resists the tape sliding. These force’s respective potentials will follow what we described in section 4. We will set up the notation in subsections 10.1 through 10.3, and then define $D_t(q)$ and $D_n(q)$ in subsection 10.4, and then we will discuss their respective forces in subsections 10.5 and 10.6.

10.1 Configuration Space and Selection Matrices

For the calculation of these forces that take place between an adhesive body and some other body we will calculate the force per face $f$ and use a reduced configuration space $q$. We will denote the positions of the three vertices of $f$ as $p_1$, $p_2$, and $p_3$. If the face is stuck to a static body the configuration space is a vector in $\mathbb{R}^9$ such that

$$qs = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

If the face is stuck to a rigid body with translation $\tau$ and axis-angle representation $\theta$ then we let the configuration space be a vector in $\mathbb{R}^{15}$ such that

$$qR = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \tau \\ \theta \end{bmatrix}$$
If the face is stuck to an adhesive body then it is stuck to a specific face $g$ on that adhesive body. Let $r_1$, $r_2$, and $r_3$ be the positions of the three points of $g$. Then we define the configuration space as a vector in $\mathbb{R}^{18}$

$$q_A = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

We will also be using $S_{p_1}$, $S_{p_2}$, $S_{p_3}$, $S_r$, $S_\theta$, $S_{r_1}$, $S_{r_2}$, and $S_{r_3}$ to denote the selection matrices that take in $q$ and return the corresponding vector in $\mathbb{R}^3$. We will also use the shorthand notation that $p_1 = S_{p_1}q$ and similarly for $p_2$, $p_3$, $\tau$, $\theta$, $r_1$, $r_2$, and $r_3$.

### 10.2 Tape Face Calculations

It will be helpful for us later on if we cover some common calculations here. To find the centroid of a stuck tape face with configuration $q$ we use the following function:

$$C(q) = \frac{1}{3}(p_1 + p_2 + p_3)$$

We will also want the differential of this function which is defined as:

$$[dC] = \frac{1}{3}(S_{p_1} + S_{p_2} + S_{p_3})$$

To help with later definitions we will define the hat function that takes in a vector $v$ and returns that vector normalized:

$$H(v) = \frac{v}{||v||}$$

and its corresponding differential:

$$dH(v) = \frac{I}{||v||} - \frac{vv^\top}{||v||^3}$$

Another item that will be useful for us is calculating the normal of the side of the tape face that is stuck. We let the parameter $\alpha \in \{-1, 1\}$ denote which side of the tape face is stuck and then we can calculate the normal of that side from the configuration space $q$ with the following function:

$$N_t(q) = \alpha H((p_2 - p_1) \times (p_3 - p_1))$$

Letting $v_x$ represent the cross product matrix for a vector in $\mathbb{R}^3$ we define the differential as follows:

$$[dN_t] = \alpha \{dH((p_2 - p_1) \times (p_3 - p_1))\}(-(p_3 - p_1)_x(S_{p_2} - S_{p_1}) + (p_2 - p_1)_x(S_{p_3} - S_{p_1}))$$
10.3 Stuck Point and Stuck Normal

The other big factor in determining how the adhesive and slide resistance should work is the point that the tape face is stuck to. Let us say we are stuck to a point $s$ in local coordinates. We will show how to calculate the world position of the point $s$. We note in the case of a static body we have that:

$$\Phi_s(q) = s$$

which makes sense since the stuck position in this case does not vary with small perturbations of the configuration. In the case of a rigid body we have that:

$$\Phi_r(q) = rot(\theta)s + \tau$$

For the last case where we are stuck to an adhesive body then our local coordinates are a face index $i$ and barycentric coordinates $\bar{v}$ then we note we can get the world coordinates of $s$ with:

$$\Phi_a(q) = [r_1, r_2, r_3]\bar{v}$$

We will also make note of their differentials since they will be extremely useful in future derivations.

$$[d\Phi_s] = 0$$
$$[d\Phi_r] = \{drot(\theta)\}(s)S_\theta + S_\tau$$
$$[d\Phi_a] = \bar{v}_1S_{r_1} + \bar{v}_2S_{r_2} + \bar{v}_3S_{r_3}$$

We will also want to know the stuck normal in global coordinates. In the case of a static or rigid body let us say we initially recorded that we were stuck with normal $n$ in local coordinates then we have that our global stuck normal is

$$M_s(q) = n$$
$$M_r(q) = rot(\theta)n$$

We also have to account for the adhesive case, we will let $\sigma$ be 1 if we are stuck to the front of the other face and $-1$ if we are stuck to the back of the other face. Then we note the normal of the face we are stuck to can be calculated as follows

$$M_a(q) = \sigma H((r_2 - r_1) \times (r_3 - r_1))$$

Just like for the stuck position formulas it will be useful for us to make note of their differentials:

$$[dM_s] = 0$$
$$[dM_r] = \{drot(\theta)\}(s)S_\theta$$
$$[dM_a] = \sigma\{dH((r_2 - r_1) \times (r_3 - r_1))\}(-(r_3 - r_1) \times (S_{r_2} - S_{r_1}) + (r_2 - r_1) \times (S_{r_3} - S_{r_1}))$$

We have defined these functions with subscripts but will just use $\Phi$ and $M$ as a shorthand for these functions and the appropriate subscript should be chosen based on which face-face interaction is happening.
10.4 Center to Stuck Point Difference

We note when a tape face is stuck to a face of another body it will be important for us to know the vector between the two points of the adhesive bond. Let \( \bar{s} \) and \( \bar{n} \) be the local stuck point and local normal and \( s \) and \( n \) be their global counterparts and \( q \) be the configuration defined as before, then we can define the difference vector as follows:

\[
D(q) = C(q) - \Phi(q)
\]

The differential of this difference vector is:

\[
[dD] = dC - d\Phi
\]

It will be important to know the component along \( n \) which we will denote as \( D_n \) and the difference vector projected onto the tangent plane of the stuck face which we will denote as \( D_t \). We can write \( D_n \) as follows:

\[
D_n(q) = M(q) \cdot D(q)
\]

which has the following differential:

\[
dD_n(q) = M(q)^T[dD] + D(q)^T[dM]
\]

Then we note the part of the difference vector lying on the tangent plane can be retrieved by removing the component in the normal direction. Therefore we note the formula for \( D_t \) is:

\[
D_t(q) = D(q) - D_n(q)M(q)
\]

which has the following differential:

\[
dD_t(q) = dD(q) - (M(q)[dD_n] + D_n(q)[dM])
\]

Another important quantity closely related to \( D_t(q) \) that we will be making use of is \( ||D_t(q)||^2 = D_t(q) \cdot D_t(q) \) which we will denote as \( J \). We note that if we let:

\[
J(q) = D_t(q) \cdot D_t(q)
\]

then we can find that the differential of \( J \) is

\[
[dJ] = 2D_t(q)^T[dD_t]
\]

10.5 Slide Resist Potential and Force

In this section we will discuss the derivation of our slide resist potential and its respective force. When a tape face with area \( A \) is stuck to a surface it resists sliding along this surface, we want to create a potential that is minimized when the center of the tape’s face lies directly over the point it is stuck to. We will also allow the user to define their own friction coefficient \( \zeta \) to scale how much the tape resists sliding. To achieve this effect we use a spring-like potential on the \( D_t(q) \) which we define as:

\[
V_{\text{slide}}(q) = \zeta AJ(q)
\]
This potential is zero when \( C(q) \) lies directly over \( \Phi(q) \) and is positive when this is not the case since \( D_t(q) \) will have some positive length. We note we factor the area into this equation since subdividing the mesh should have no effect on the overall slide resist energy of the mesh beyond just being a refinement of the estimate provided by a coarse mesh. The differential of this potential is simply just:

\[
[dV_{slide}] = \zeta A \,[dJ] 
\]

and since force is just the negative of the differential of the potential we note we have that:

\[
F_{slide} = -\zeta A \,[dJ] 
\]

### 10.6 Adhesive Potential and Force

We will discuss our choice of adhesive potential in this section and its corresponding differential and force. Intuitively we want this potential to be minimized when the tape face is close to the stuck face and the two faces are well-aligned therefore maximizing the shared contact region between the two. We have already discussed a way for getting an approximation of the closeness of the faces with \( D_n(q) \) so our next step should be finding a way to approximate the contact area between the two faces. We will do this approximation by finding the area of the tape face’s shadow when projected onto the stuck face. We note if the original face has area \( A \) and the corresponding face normals are \( N_t(q) \) and \( M(q) \) then we have that the area of the shadow is:

\[
A_{shadow}(q) = -A(N_t(q) \cdot M(q)) 
\]

We note this area is maximized when \( N_t(q) = -M(q) \) and is 0 whenever \( N_t(q) \) and \( M(q) \) are orthogonal. One might worry that this value is negative whenever \( N_t(q) \) and \( M(q) \) roughly point in the same direction but due to our detachment criteria a face will never be stuck in this case. It is also important for us to know how the shadow area changes as the configuration changes and therefore we will want the differential of \( A_{shadow} \) which is:

\[
[dA_{shadow}] = -A(M(q)^T[dN_t] + N_t(q)^T[dM]) 
\]

We will then briefly forget about \( A_{shadow} \) and try to create a potential that is just with the closeness of the faces. We want the adhesive force not to affect bodies that are far away and we want a sharp drop to the rest tape since tape seems to stick almost instantaneously. Letting \( \gamma \) be a user-defined constant affecting how sticky the tape is and \( r_m \) be the rest distance for the tape the first potential for closeness that we tried was the following:

\[
\gamma((\frac{r_m}{D_n(q)})^{12} - 2(\frac{r_m}{D_n(q)})^6) 
\]
This potential is just a scaled version of Lennard-Jones potential on the distance in the normal direction. We note this formulation has the quality that the potential reaches a minimum at \( r_m \) with a value of \( \gamma \). However, there are two problems that this potential would present: the function does not approach zero quickly as the objects get far apart and the potential quickly increases up above zero as the distance in the normal direction approaches zero. We note the latter problem would lead to problems later on when we try to scale this closeness potential by the area of the region leading to behavior where the tape turning away from the region it is trying to stick to would lower the potential. We therefore change the formulation above to scale up faster and by changing the potential to act like a simple quadratic when closer than \( r_m \):

\[
V_{\text{closeness}}(q) = \begin{cases} 
\gamma(\exp(-36(D_n(q) - 1)) - 2\exp(-18(D_n(q) - 1))) & D_n(q) \geq r_m \\
\gamma \left( \frac{1}{r_m} \left( D_n(q)^2 - 2r_mD_n(q) \right) \right) & D_n(q) < r_m 
\end{cases}
\]

Original Closeness Formulation with \( \gamma = 1 \) and \( r_m = 1 \)

\[ V_{\text{closeness}} \text{ with } \gamma = 1 \text{ and } r_m = 1 \]

This scaling change gives the tape a much sharper region where it sticks to the surface instead of partially adhering from far away. We note that this potential reaches a minimum of \(-\gamma\) when the distance between the faces is \( r_m \), we note for \( D_n(q) > 0 \) it is the case that \( V_{\text{closeness}}(q) \) is always negative. This is desirable since we will want the property that by multiplying the value of the function at a fixed \( D_n(q) \) by a value in the range \([0, 1]\) we achieve the minimum at that \( D_n(q) \) by multiplying by 1. We also note that there is a steep potential drop as \( D_n(q) \) approaches \( r_m \) from the right representing the tape face wanting to go to the rest distance. We note that this is just our formulation from section 4 without the area contribution and with the following choice of \( G \) to represent the closeness of two faces:

\[
G(x) = \begin{cases} 
2\exp(-18(x/r_m - 1)) - \exp(-36(x/r_m - 1)) & x \geq r_m \\
\frac{1}{r_m^2}(2r_mx - x^2) & x < r_m 
\end{cases}
\]

The differential of \( V_{\text{closeness}} \) will be useful to us so we will make note of it below:

\[
[dV_{\text{closeness}}] = \begin{cases} 
\frac{36\gamma}{r_m^2}(\exp(-18(D_n(q)/r_m - 1)) - \exp(-36(D_n(q)/r_m - 1)))dD_n & D_n(q) \geq r_m \\
\frac{\gamma}{r_m^2}(2D_n(q)dD_n - 2r_mdD_n) & D_n(q) < r_m 
\end{cases}
\]
It is useful to note that both parts of the piecewise function are equal to zero when $D_n(q) = r_m$ meaning that the differential is continuous since the two pieces are both continuous and the ends meet up.

We then note that we can write a potential that takes into account both $A_{\text{shadow}}$ and $V_{\text{closeness}}$ into account by taking the product of the two which roughly estimates the potential of all the points on the tape face. We will denote this potential as $V_{\text{adhesive}}$ and note that it can be written as:

$$V_{\text{adhesive}}(q) = A_{\text{shadow}}(q)V_{\text{closeness}}(q)$$

We then will want the differential of $V_{\text{adhesive}}$ which can be achieved simply using the product rule on the above to give us:

$$[dV_{\text{adhesive}}] = A_{\text{shadow}}(q)[dV_{\text{closeness}}] + V_{\text{closeness}}(q)[dA_{\text{shadow}}]$$

Since the force is just the negative of the differential of the potential for a conservative potential we note then that we have that $F_{\text{adhesive}}$ can be defined as:

$$F_{\text{adhesive}} = -A_{\text{shadow}}(q)[dV_{\text{closeness}}] - V_{\text{closeness}}(q)[dA_{\text{shadow}}]$$

11 Cloth Interactions

In this section, we will discuss the model we chose to model the cloth interactions. These cloth interactions will allow a side of a piece of tape that is not sticky to be modelled in a reasonable way. Additionally, for a sticky side of a piece of tape it will act as a fallback in case our tape forces are not strong enough to keep an object from penetrating through the tape. First, we create a bounding volume hierarchy that encompasses all of the faces in the simulation. After calculating all of the velocities to use for the next time step, we do a collision check for each vertex in the mesh. Letting $r_m$ denote a piece of tape’s preferred distance to the surface it is adhered to, we will apply a collision impulse if the vertex comes within $\frac{3r_m}{4}$ units of a face and the vertex is approaching the face at any point within a time step. We use the impulses given by ”Robust Treatment of Collisions, Contact and Friction for Cloth Animation” to keep cloth from phasing through objects. [3] We will deal with two types of cases: vertex-face collision between two pieces of cloth and vertex-face collision between cloth and a rigid body.

If we find that a vertex $t$ of a piece of cloth will come within $\frac{3r_m}{4}$ units of another cloth face $f$, we will apply an impulse to keep the collision from happening. The impulse will be along the direction of the normal of $f$ and will have magnitude just large enough to cancel out their relative velocities. We now want to find the magnitude $\alpha$ that will exactly cancel out the two velocities. Letting $v_n$ be the relative velocity between $t$ and $f$ in the direction of $n$ we will only apply an impulse if $v_n < 0$ meaning that $t$ and $f$ are approaching each other. If we let $m_t$ be the mass of the vertex $t$, $m_1$, $m_2$, and $m_3$ be the masses of the
vertices of the face $f$ along with letting $w_1$, $w_2$, and $w_3$ be the barycentric coordinates of the point on $f$ that is closest to $t$ we note we can derive that:

$$\alpha = \frac{-v_n}{m_t + \frac{w_1^2}{m_1} + \frac{w_2^2}{m_2} + \frac{w_3^2}{m_3}}$$

We then just need to apply an impulse of $-w_i\alpha n$ for each vertex $v_i$ on face $f$ and an impulse of $\alpha n$ for vertex $t$.

When we find that a vertex $t$ of a piece of cloth will come within $\frac{3\text{rm}}{4}$ units of a rigid body face $f$, we will apply an impulse to keep the collision from happening. Let $\tau$ and $\theta$ be the configuration of the rigid body when the collision is detected, $\dot{\tau}$ be the rigid body’s translational velocity and $\omega$ be the rigid body’s angular velocity. Let $\bar{p}$ be the closest point on $f$ to $v$ in the local coordinates of the rigid body with $p$ being the corresponding position in global coordinates such that formally we have that $p = \phi_p(\bar{p})$. We will also define $n$ to be the global normal of the face $f$, we can calculate $n$ by taking the local normal of the face $\bar{n}$ and calculating $\phi_v(\bar{n})$. We will denote the mass of $t$ as $m_t$, the mass of the rigid body that $p$ lies on as $m_p$, the inertia tensor for the rigid body that $p$ lies on as $M_I$, the velocity of $t$ as $v_t$ and the velocity of $p$ as $v_p$, and we will let $v_n$ be the relative velocity between $v$ and $p$. We will apply an impulse only if $v_n < 0$. The magnitude of the impulse $\alpha$ should be:

$$\alpha = \frac{-(v_t + \text{rot}(\theta)[\bar{p}]\times\omega - \dot{\tau})\cdot n}{(\frac{n}{m_t} - \text{rot}(\theta)[\bar{p}]\times M_I^{-1}[\bar{p}]\times\text{rot}(-\theta)n + \frac{n}{m_p})\cdot n}$$

We then apply an impulse of $\alpha n$ to $t$ and an impulse of $-\alpha n$ to the rigid body $p$ lies on.

12 Test Scenarios

In the following subsections we will show off test scenarios with our tape model that shows how it interacts to different stimuli. We will use a color coding in our visuals to help readers understand what is happening in the diagrams. Grey indicates a sticky side of an adhesive body while purple indicates that a side of an adhesive body that does not stick. Objects that are colored green are rigid bodies and those that are blue are static bodies.

12.1 Tape Bridge Test

The Tape Bridge Test involves creating a bridge between two heavy static blocks to catch a rigid body ball. If the ball is light relative to the strength of the adhesive of the tape, the ball is expected to fall and come to rest on the tape bridge; however, if the ball is heavy relative to the strength of the adhesive the ball will cause the tape to separate from the boxes and both the ball and the tape bridge should fall.

In the first simulation we let $\gamma = 1000$ and $\zeta = 50000$ with a simulation time step of $5 \times 10^{-5}$ and letting our cloth’s minimum distance be 0.01.
We note in the simulation above the ball hits the tape surface and gets stuck to it. The tape slides off from the top of the blocks and starts to slide down the inside of the static blocks. The tape bridge eventually comes to rest with the ends of the tape bridge attached to the sides of the static block.

For our second simulation we leave all the parameters the same except we scale $\zeta$ by a factor of ten such that $\zeta = 500000$:
In this simulation, we note the ends of the tape bridge do not ever slide far from their initial position. The rigid ball gets stuck to the tape bridge and bounces in place.

Finally, we also ran a simulation with the same settings as the two above except that we let $\zeta = 5000$ and $\gamma = 500$ to make the tape weaker.

We note in this simulation the tape has almost no sticking power and therefore does not support the rigid ball. They both fall to the ground and the rigid ball bounces back up due to not being strongly adhered to the tape.

12.2 Tape Pendulum Test

The Tape Pendulum Test involved simulating a piece of double-sided tape that is vertical and has its top stuck to a static body cube. A rigid body cube is thrown at the piece of tape and we should be able to observe behavior where the rigid body sticks onto the tape and swings on it like a pendulum. The rigid body should also rotate so that its top generally stays pointing towards the pendulum’s pivot point.
We note in the above simulation we have put a checkered pattern over the green ball so it is easier to see its alignment. We note it hits the tape and rises slightly while rotating along with the tape. The rigid ball eventually loses speed and starts to fall back to center.

12.3 Alternate Tape Pendulum Test

This Tape Pendulum Test involves simulating a piece of double-sided tape that is looped around a static bar. A rigid body sphere is thrown at the piece of tape and we should be able to observe behavior where the rigid body sticks onto the tape and swings on it like a pendulum. The rigid body should also rotate so that its top generally stays pointing towards the pendulum’s pivot point. We run a simulation where the tape has parameters $\gamma = 500$ and $\zeta = 50000$: 
We note the tape rises to above horizontal with the bar due to a high initial velocity. Looking closely at the diagram, we can see that the rigid ball slides down the length of the tape a bit after initially sticking to the tape. Next, the ball loses momentum and starts to pull in a bit and swing back to center. This overall matches what we would expect of a pendulum.

12.4 Tape Twist Test

The Tape Twist Test involves twisting the tape by fixing one edge to be stuck to a solid surface while fixing the other edge to be stuck to a rotating wheel. If the wheel is spinning
with enough force and the tape’s adhesive strength is low enough we expect after enough twisting for the tape to detach; however, if the force of the spinning wheel is low compared to the tape’s adhesive strength we expect that the tape will force the wheel to stop spinning. In the first simulation we let $\gamma = 1000$ and $\zeta = 500000$ with a simulation time step of $5 \times 10^{-5}$ and letting our cloth’s minimum distance be 0.03.

We note that the rigid spheres start to approach each other as the cloth coils up which makes sense since the tape has less available material to fully stretch out with. The rigid sphere starts to slow down as the bending of the tape applies torque to them and near the end they start to spin in the opposite direction to uncoil the tape. If we run the simulation again but this time letting $\gamma = 100$ and $\zeta = 50000$ we get the following.
We note that the tape does not stick to the spheres and instead just gains a bit of spin and bend from its contact. The tape begins to bend back to center as time progresses.

12.5 Tape Roll Test

The Tape Roll Test involves having a rigid body cylinder start rolling onto a piece of one-sided tape with the sticky side up. The cylinder should wrap the tape around it as it rolls down the length of the tape. For this simulation we let $\gamma = 500$ and $\zeta = 50000$. 
We note that in the above simulation that the tape starts to coil around the cylinder. The tape coil eventually grows thick enough to lift the cylinder off the ground and cause it to wobble as it continues rolling the rest of the way.

12.6 Tape Fold Test

The Tape Fold test involves dropping an adhesive body onto a static sphere and observing how it folds over the sphere. We start with a fine mesh for our adhesive body with a time step of \(5 \times 10^{-5}\), \(\gamma = 100\), and \(\zeta = 50000\).
In the above simulation, we can see that the tape folds over the sphere much like a piece of cloth would. If we rerun the simulation with $\gamma = 1000$ and $\zeta = 500000$ we get the following:

In this simulation, the adhesive force is stronger so the tape takes on a fold more like a bow-tie such that it covers more surface area of the static sphere.
12.7 Tape Sandwich Test

The Tape Sandwich test involves a piece of double-sided tape with two pieces of cloth on opposite sides of the tape coming towards the tape in the center. The two pieces of cloth have velocities that make it such that they do not hit the tape dead center. We ran the simulation with $\gamma = 100$ and $\zeta = 50000$:

We note in the above diagram that the three pieces clump together and fall to the ground. When they hit the ground the piece of tape that is exposed to the ground sticks and stay mostly still while the cloth exposed to the ground continues to bounce a bit.

12.8 Tape Window Test

The Tape Window Test involves adhering a square piece of tape over a static window frame. We then launch a tilted rigid body cube into the tape. If the tape is sticky enough, we expect it to stick and bounce with the window. If instead the tape is not sticky enough, we expect it to peel off the tape.

In the first simulation we let $\gamma = 100$ and $\zeta = 50000$ with a simulation time step of $5 \times 10^{-5}$ and letting our cloth’s minimum distance be 0.03.
We note that when the cube hits the tape surface it peels the tape off of the window frame and the tape sticks lightly to the surface of the cube. The cube and tape then fall together and land on the ground with the underside of the tape sticking to the ground.

For our second simulation we leave the simulation settings the same except we scale $\gamma$ and $\zeta$ by a factor of ten such that $\gamma = 1000$ and $\zeta = 500000$.

We note in this simulation the rigid body cube is not strong enough to peel the tape off
the window frame. The cube hits the tape and gets stuck to it and over time as the tape stops bouncing back and forth the cube aligns itself so that its face lies flat against the tape.

13 Future Work

There are a few things we would have liked to experiment more with the model given more time. First, we believe combining the slide potential and adhesive potential in a more natural way such that they were interlocked better would be useful. For example, a slide force that decreases as a tape face gets further away from the face it is stuck to might lead to improvements in realism. We also require that all of the rigid and static bodies not have sharp corners. Objects with sharp corners can cause a quick change in the normal distance when changing which face you are stuck to. An interesting idea would be modifying the method to smooth out these sharp edges in computation. Finally, we note adding in edge-edge collision detection for our tape would keep the edges of our adhesive bodies from phasing into the edges of other adhesive bodies.

References


